UDC 510.67:512.56

GRAPH MODELS OF COMPLEX NETWORKS

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Abstract. The study of the properties of analytical, social and economic networks marked the beginning of the study of complex networks in general. Typical studies of the network, the models of which are graphs, answered questions of centering and connectivity, finding the shortest paths, cycles or trees, minimal covers, or maximal matching.
As one or another peak, an economic agent or person in the network in question is connected with another peak, an economic agent or another person, which or who of them has the most connections with others. The term “complex networks” appeared at the beginning of this century and refers to networks with a more complex architecture than classical networks with a given number of nodes and connections. Usually in such networks it is communications that largely determine their properties.

Keywords: analytical, social and economic networks, complex networks, weighted location problem, centers, graph model, shortest paths, cycles or trees, minimal covers, or maximal matching

The study of complex networks is concerned with mathematics, economics, sociology, information technology, digital economics, biology, physics, and chemistry, which, in turn, makes it possible to assign complex networks to an interdisciplinary area of science. An example of such tasks is the spread of infection (epidemic), studied in medicine, which is reflected in information technology, as the process of spreading viruses on the Internet, or as the dissemination of fake information in social networks or media.

The situation with the study of complex networks is of general interest also because we are witnessing a “happy for scientists” phenomenon — the practice in this area is far ahead of theory. This means the emergence of an enormous number of new tasks that require generalization and immediate solution, both in practice and new theories.

Historically, the study of complex networks began with their practical study, search for any properties, patterns, confirmation or refutation of the existence of these patterns. One of the main properties of analytical, social and economic networks discovered in this way was the phenomenon of the small world (the small - world phenomenon). Its main idea: often the economic and social world consists of short links of interactions between economic agents (companies, households, power structures) and social objects. Thus, the main task of researchers is to obtain a model that corresponds to certain properties of real networks.

The first model of the small world (graph model) was proposed by Watts and Strogatz (Watts and Strogatz), in which the effect of the small world was explained as follows: large networks tend to have a small diameter and a small average length of the geodesic circuit.

The second important idea arising from the study of economic and social networks was that they tend to have a high clustering coefficient. The idea of clustering is important in analytics, economics, social interactions where they pay attention to regular (homogeneous graphs) interactions that make complex networks resistant to internal and external disturbances (challenges), for example, triangles (three mutually connected vertices) in analytical, social and economic networks.

Many complex networks have the property of homophilia, which reflects the fact that economic agents, social communities (networks) are more inclined to enter into relationships with similar groups.

Another important observation in complex networks is the conclusion that weak links play a connecting role in interactions between groups, while strong links are responsible for cohesion within communities of economic agents, social objects and increase the overall separation in the economy and society.

The first models that tried to isolate the types of graphs that correspond to the properties and structures of real-world networks came from classical graph theory and assumed that networks are formed randomly.

Random graphs can be generated in a variety of ways so that the models built on their basis better correspond to real complex networks. One of such graph generation models is a configuration model that allows you to create networks with a precisely defined distribution of vertex degrees (degree distribution), which in turn allows you to generate different networks with the same distribution as the complex network under study.

In contrast to stationary networks, there is a special class of random graphs in which a new vertex appears over time and, when it appears, forms edges with existing vertices. The main purpose of creating such models was an attempt to describe models of the Internet, where the new page creates links to already existing pages. Similar models can be described and other economic and social networks. For example, establishing new acquaintances when a person comes to a new school, is hired for a job, or gets acquainted with neighbors after moving or creating new supply chains in economic networks.

The next steps in trying to explain the growth phenomena of complex networks were the creation of algorithms for their generation based on the description of the growth of scale-free graphs, including Erdës-Rényi graphs (P. Erd Ers, A Rényi) and graphs generated in accordance with the Barabash-Albert model (Albert- László Barabási & Réka Albert, etc. This approach took into account two important general concepts found in real complex networks, the growth of a graph and the principle of preferred connection.
In modern growth models of complex networks, for example, formulated in the works of Ozik and Hunt (Ozik J., Hunt B.), Peng and Zang (Aoyuan Peng, Lianming Zhang), the algorithm of deterministic multidimensional growth (DMG) of the “small world” graph .
At this point in time, there are many deterministic models, like DMG models or geometric assortative growth models, suitable for generating “small world” graphs, which can be used to simulate the growth dynamics of real-world networks, but they are not exhaustive and need further development.
Thus, the existing growth algorithms for small world graphs, despite their deterministic character, make it possible to model the growth of networks that are somewhat similar to real ones, but they take into account the so-called “weak” links to a lesser extent than the latter are actually found in modern analytical, economic and social networks. Note also that not all economic agents and their groups, as well as the communities of modern complex networks fully comply with the definition of a “small world”.
In connection with the rapid development of complex networks, the digital economy, the emergence of new practical problems and the lag of their theoretical support, the development of new models and algorithms that explain and predict the behavior of economic agents (companies, households, power structures) and social communities to understand their goals and predict their future.

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