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GRAPH MODEL OF THE PHARMACEUTICAL ORGANIZATION'S NETWORK LOCATION IN THE CITY

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Abstract. In this paper, we developed a method for an optimal location of the pharmacy network by the example of Vladivostok city. Method is based on the weighted p-center location problem. The weights in the problem are determined using econometric and statistical methods that can take into account the specifics of placed objects.

Keywords: weighted location problem, p-centers, discrete optimization, pharmaceuticals, graph model.

Location problems are aimed at solving one of the major modern issues - minimizing time expenses. The main purpose of the proper trade facilities location is to meet the needs of the population in high-speed provision of services. For an optimal service points' location, a number of factors, specific to a particular type of service, must be considered. Successful entrepreneurship is characterized by a large number of pharmaceutical companies wishing to cooperate. To attract partners it is necessary to have a high turnover, which depends on the correct location of pharmacies. Not only profit, but also the minimization of the time spent on the purchase of necessary medicines by the population depends on the best location of pharmacies. A person's life may depend on the service rapidity, which is why it is necessary to minimize the maximum distance from potential consumers to a trade object. In order to solve this type of problems there are corresponding location theory methods [1].

The description of the problem of the location of pharmaceutical organizations is based on the theory of pharmaceutical marketing, and the technical part is based on the theory of extreme problems on networks and graphs, methods for solving problems of discrete location, econometric theory and statistics.

Over the past decades, location problem has become a topic of high interest in the fields of continuous and discrete optimization research. In these mathematical models, a set of objects (e.g. resource) is located in a way to minimize the costs of satisfying the consumer according to a certain set of restrictions.

During the determination of pharmacy organization's best location, it is necessary to minimize the maximum distance from potential consumers to a trade object. For such a problem, the most suitable graph model is the p-center problem, since in this model it is necessary to find a minimum that covers the distance that covers each vertex of the graph. Below we provide an overview of the main results in a p-center location theory.

S. L. Hakimi suggested the first solution for the absolute center problem in 1964. In his article he showed that the optimal location of the center is always located in the vertex of the graph (network node), and the best position of an absolute center does not always match with the vertex [2]. He also proposed a solution algorithm for an absolute graph centers. The method offered by S. L. Hakimi is a graphical since the determination of optimal candidate-points occurs due to a comparison on charts distance point-to-vertex for different vertices.

Furthermore the Hakimi method is intended for solving the problem for one absolute center and cannot be generalized to the case of absolute p-centers. To determine p-centers S. Singer suggested application of heuristic methods to find approximate solutions and Minieka E. applied the solution of covering problem for p-centers model. [3, 4].

Nicos C. and Peter V. offered a non-graphical method that uses an iterative algorithm for solving the problem of absolute p-centers. They showed that this method is a fast converging as search of absolute center can be finished immediately as soon as the required precision in the location of the center is reached, and this method can be modified so that it was possible to find solutions that are close to the optimum [5].

In 1985 Dyer and Frieze have proposed the greedy algorithm where the first center is randomly selected. Hochbaum in the same year introduced a dual approximate heuristic approach to solve the p-center problem [6].

In 1998 Anderson and others presented an approximation algorithm based on the demand in a set of points (demand point aggregation method) [7].

In 2000 Khuller and Sussmann offered the approximate algorithm with a limited capacity [8]. Pallottino and others in the same year published a local heuristic approach for solution of p-centrum problem where vertices have limited capacity [9].

Burkard and Dollani introduced p-centers model with positive or negative weights. It is a model where vertices can have positive or negative weight, which determines if an object favorable or not [10].

In 2003 Caruso and others have presented four algorithms. They found a solution to p-center problem, based on the solution of a sequence of covering problems. Two heuristic algorithms can solve the problem with 900 vertices in a few seconds [11].

Nowadays there are many studies on the problems of locating objects, for example such as points of public services. In these problems the potential demand for services of placing objects is considered as vertex weights. In this research, we consider as weights the estimate of pharmaceutical companies' turnover.

Consider the problem of one city district's population service. Crossroads can be presented as graph vertices and roads as edges having a weight equal to the path length between corresponding crossroads. The required number of pharmacies must be placed at crossroads in such a way that the travel time from pharmacy to the farthest vertex of this graph would be minimal.

Location model was considered as a problem of integer linear programming. Assumptions of the model: allocation objects can only be placed on the vertices. Let such vertices to be called the candidate-vertices. It is necessary to locate *p* objects, fixed objects, requiring the services of allocation objects are on vertices. Let such vertices to be called customer-vertices, those vertices have weights.

For the solution of this problem it is possible to apply a heuristic method of Lagrangian relaxations. Hence the problem should be represented in the form of integer linear programming problem.

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Exogenous parameters: d_{ij} - the length of the shortest path between the customer-vertex i and the candidate-vertex j; p - the number of objects to be allocated; h_i - the customer-vertex *i* weight. Endogenous parameters: $X_j = 1$ if an object is located in the candidate-vertex j, otherwise 0. Distribution matrix: $Y_{ij} = 1$ if the customer-vertex i is attached to the candidate-vertex j, otherwise 0. Maximum distance *z* between the customer-vertex and the closest object.

The objective function of the problem:

$$\min z \qquad (1.1)$$

Boundaries:

$$\sum_j Y_{ij} = 1 \;\; \forall i, \qquad (1.2)$$

$$\sum_j X_j = p, \qquad (1.3)$$

$$Y_{ij} \le X_j \;\;\; \forall i, j \;\;\; , \qquad (1.4)$$

$$z \ge h_i \sum_j d_{ij} Y_{ij} \;\;\; \forall i \,, \qquad (1.5)$$

$$X_j \in \{0,1\} \;\; \forall j \,, \qquad (1.6)$$

$$Y_{ij} \in \{0,1\} \;\; \forall i, j \;. \qquad (1.7)$$

Equation (1.1) in combination with (1.5) minimizes the maximum distance between the customer-vertex and the closest object. Equation (1.2) states that exactly one object is assigned to all customer-vertices. Equation (1.3) ensures the location of p objects. Inequality (1.4) provides a link between the location variables and the distribution variables. Boundaries (1.6) - (1.7) ensure that the location variables and the distribution variables are binary [4].

In order to solve the considered location problem, the heuristic method of Lagrange relaxation was applied. Many NP-difficult problems enable us to separate the system of boundaries into two groups so that the removal of one of them turns a problem into a polynomially solvable problem. These limits are entered into the objective function with certain coefficients, Lagrange multipliers. In such a way relaxed problem gives a lower bound on the optimum of the original problem, if it is a minimization problem, and the upper bound for the maximization problem.

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