On one problem of optimal control

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We consider the problem of optimal control in the form

$$J = \int_{t_0}^{t_1} (c_0 + c_1 x_1 + c_2 x_2 + ... + c_n x_n + \alpha u^2) dt \to \min ,$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ ... \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & ... & A_{1n} \\ A_{21} & A_{22} & ... & A_{2n} \\ ... & ... & ... & ... \\ A_{n1} & A_{n2} & ... & A_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \\ ... \\ B_n \end{pmatrix} u , \qquad (1)$$

$$\begin{pmatrix} x_1(t_0) \\ x_2(t_0) \\ ... \\ x_n(t_0) \end{pmatrix} = \begin{pmatrix} x_1^0 \\ x_2^0 \\ ... \\ x_n^0 \end{pmatrix}, \quad \begin{pmatrix} x_1(t_1) \\ x_2(t_1) \\ ... \\ x_n(t_1) \end{pmatrix} = \begin{pmatrix} x_1^1 \\ x_2^1 \\ ... \\ x_n^1 \end{pmatrix}, \ u \in U \subseteq R,$$

where $u(t)$ is control variable, $x(t) \in R^n$ is state variable, $x^0 = \begin{pmatrix} x_1^0 \\ x_2^0 \\ ... \\ x_n^0 \end{pmatrix}$, $x^1 = \begin{pmatrix} x_1^1 \\ x_2^1 \\ ... \\ x_n^1 \end{pmatrix}$ are fixed ends of trajectory, c_1, c_2 , … , c_n and α are constants, t_0, t_1 are fixed moments of time

If $c = 0$ minimum of functional in problem (1) corresponds to minimum of energy spending on control. Problem (1) is its natural generalization allowing to form and solve the variety of real problems.

According to classification [1] problem (1) is the General problem of optimal control and its solution – process $x(t)$, $u(t)$, satisfies Pontryagin maximum principle developed by L.Aschepkov, D. Dolgy, etc. for this type of optimal control problem. In general case of mathematical model of the General problem of optimal control the procedure of obtaining the exact solution on the base of maximum principle is very complicated or impossible. The most of numerical methods - Newton's method, Gradient method and others, can give an approximate solution with some accuracy depending of many factors. And convergence of these methods plays very important role. In particular, there is a bad convergence of initial approximation for the values of conjugate variables to the values that put zeros for residual functions because of permanent getting by them their local minimum. This leads to that Newton's method or Gradient method cannot give a good result.

In this research, we introduce a new approach for solution of the problem (1) based on the successive using Pontryagin maximum principle and Galerkin method. We show that proper choice of trial functions in Galerkin method can give an exact solution of the problem (1). This significant advantage of Galerkin method can be generalized on the other types of optimal control problems.

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2010 MATHEMATICS SABJECT CLASSIFICATION. 34K35, 47N70, 93-XX.

KEYWORDS AND PHRASES. Optimal control problem, Pontryagin maximum principle, Galerkin method, residual function, trial function.

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