UDC 340.5

Name and surname of the author/authors,
Graduate School of Education, Konkuk University, Lee-Chae Jang
E-mail: Lcjang@konkuk.ac.kr

A note on some properties of fully degenerate Bernoulli polynomials associated with degenerate Bernstein polynomials

Abstract. In this paper, we investigate some propoties and identities for fully degenerate Bernoulli polynomials in connection with degenerate Bernstein polynomials by means of bosonic p-adic integrals on $\Bbb Z_p$ and generating functions. Furthermore, we study two variable degenerate Bernstein polynomials and the degenerate Bernstein operators.
Key words (degenerate Bernoulli polynomials; degenerate Bernstein operators).

Introduction and main results

Let p be a fixed prime number. Throughout this paper, Z, Z_p, Q_p and C_p, will denote the ring of rational integers, the ring of p-adic integers, the field of p-adic rational numbers and the completion of algebraic closure of Q_p, respectively. The p-adic norm $|.|_p$ is normalized as $|p|_p=1/p$.
t^n

For λ, t∈ C_p with $|\lambda|_p < p^{-1/p-1}$ and $|t|_p<1$, the degenerate Bernoulli polynomials are defined by the generating function to be

$$\frac{t}{(1+\lambda t)^{\frac{1}{\lambda}}-1}(1+\lambda t)^{\frac{x}{\lambda}} = \sum_{n=1}^{\infty} \beta_n(x|\lambda)\frac{t^n}{n!}$$

When x=0, $\beta_n(\lambda) = \beta_n(0|\lambda)$ are called the degenerate Bernoulli numbers.
Note that $\lim_{\lambda\to 0}\beta_n(x|\lambda) = B_n(x)$, where $B_n(x)$ are the ordinary Bernoulli polynomials defined by

$$\frac{t}{e^t-1}e^{xt} = \sum_{n=1}^{\infty} B_n(x)\frac{t^n}{n!}$$

and $B_n = B_n(0)$ are called the Bernoulli numbers.

Recentely, Kim-Kim introduced the degrnerate Bernstein polynomials given by

$$\frac{(x)_{k,\lambda}}{k!}(1+\lambda t)^{\frac{1-x}{\lambda}} = \sum_{n=1}^{\infty} B_{k,n}(x|\lambda)\frac{t^n}{n!}$$

Thus, we introduce the following main results.

Theorem 1 For n∈ N∪{0}, we have

$$B_n(1-x|\lambda) = (1-x)_{n,\lambda} + \sum_{m=1}^{n} B_{m,n}(x|\lambda)B_m(\lambda)\frac{(-\lambda)^{1-m}}{(m-1)!}\sum_{k=0}^{m-1}(-1)^k\binom{m-1}{k}\frac{1}{x-k\lambda}$$

Corollary 1 For $n \in \mathbb{N} \cup \{0\}$, we have

$$B_n(2|\lambda) = (2)_{n,\lambda} + \sum_{m=1}^{n} B_{m,n}(-1|\lambda) B_m(\lambda) \frac{(-\lambda)^{1-m}}{(m-1)!} \sum_{k=0}^{m-1} (-1)^{k+1} \binom{m-1}{k} B_{k,n} \frac{1}{1+k\lambda}$$

Theorem 2 For $k, n \in \mathbb{N}$, we have

$$(x)_{n,\lambda} \sum_{m=0}^{k} \binom{k}{m} (-1)^{m-k} B_n(1-x+m|\lambda) = \begin{cases} \sum_{l=0}^{n} D_l^{(k)}(x) B_{k,n-1}(x|\lambda), & \text{if } n \geq k \\ 0, & \text{if } n < k. \end{cases}$$

References

D. S. Kim, T. Kim, T. Mansour, J.-J. Seo, Fully degenerate poly-Bernoulli polynomials with a q-parameter // Filomat, 30(4) 2016, 1029-1035.

M.-S. Kim, T. Kim, B. Lee, C.-S. Ryoo, Some identities of Bernoulli numbers and polynomials associated with Bernstein polynomials, Adv. Difference Equ. 2010, Art. ID 305018, 7 pp.

T. Kim, D. S. Kim, Degenerate Bernstein polynomials, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. :RACSAM (2018), 1-8. https://doi.org/10.1007/s13398-018-0594-9.

T. Kim, D. S. Kim, Some Identities on Degenerate Bernstein and Degenerate Euler Polynomials, Mathematics 2019, 7(1), 47; https://doi.org/10.3390/math7010047.

T. Kim, D. S. Kim, G.-W. Jang, J. Kwon, A note on degenerate Bernstein polynomials, J. Inequal. Appl. **2019**:129 https://doi.org/10.1186/s13660-019-2071-1.

T. Kim, D. S. Kim, J.-J. Seo, Fully degenerate poly-Bernoulli numbers and polynomials, Open Math. 14 (2016), no. 1, 545–-556.

T. Kim, H.I. Kwon, T. Mansour, S.-H. Rim, Symmetric identities for the fully degenerate Bernoulli polynomials and degenerate Euler polynomials under symmetric group of degree n, Util. Math. 103 (2017), 61–-72.

J.-W. Park,H. K. Pak, S.-H. Rim, T. Kim, S.-H. Lee, A note on the q-Bernoulli numbers and q–Bernstein polynomials, J. Comput. Anal. Appl. 15 (2013), no. 4, 722–-729.