

SIGN K-POTENT SIGN PATTERN MATRICES THAT ALLOW K-POTENCE

Abstract. For a real number a , the sign of a is defined $+$, $-$ or 0 if a is a positive, negative or 0 , respectively. A matrix whose entries consist of just symbols $+$, $-$ and 0 is called a sign pattern matrix. If P is a property of a real matrix, then a sign pattern matrix $A=[a_{ij}]$ allow P if there is a matrix B , the signs of whose entries equal to a_{ij} , with the property P .

By the Frobenius theorem, we can prove easily that all the irreducible sign k -potent sign pattern matrices are allows k -potence. But, not all reducible sign k -potent sign pattern matrices allow k -potence.

In this paper, we provide a necessary and sufficient condition for a sign k -potent sign pattern matrix to allow k -potence. Furthermore, we show that if A is a sign k -potent sign pattern matrix, then A allows k -potence if and only if $|A|=[|a_{ij}|]$ allows k -potence.

By the definition of the multiplication of sign pattern matrix, the entries in the block submatrix of A which are induced by the Frobenius normal form does not have the opposite sign. Hence we can construct the reduced block matrix of A , denote $\text{red}(A)$. We prove that for a sign k -potent sign pattern matrix A , A allows the k -potence if and only if $\text{red}(A)$ also allows k -potence.

Key words sign pattern matrices, sign k -potent sign pattern matrix, k -potence, allow.

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