

## REMOTE SENSING PROBLEM IN THE EXISTENCE OF ACOUSTIC NOISE IN THE OCEAN

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**Abstract.** The paper considers the problem of remote sensing of the ocean by a point isotropic sound source. The inverse problem is formulated, which consists in determining the coefficient of volume scattering in a weakly scattering medium. A formula is obtained for the calculation of the received signal taking into account acoustic noise, which is caused by a distributed sound source. Computational experiments were performed to analyze the solution of the inverse problem in the presence of acoustic noise in the medium.

**Key words** radiation transfer equation, volumetric scattering, sea bottom sonar, remote sensing.

In the single scattering approximation and point source the solution of the inverse problem (remote sensing problem) is obtained in the paper [1]. In this paper authors consider the remote sensing problem taking into account acoustical noise in the ocean.

Echo signal propagates in the medium  $G \equiv \mathbb{R}^2$ . Let  $J_0$  describe a point isotropic sound source located at the origin  $\mathbf{O}$  and emitted a pulse in the initial time.  $J_1$  is the round distributed source located at the point  $r_0$ .

$$J_0(\mathbf{r}, \mathbf{k}, t) = P_0 \delta(\mathbf{r}) \delta(t), J_1(\mathbf{r}, \mathbf{k}, t) = \begin{cases} P_1, & |\mathbf{r} - r_0| < R \\ 0, & \text{иначе.} \end{cases} \quad (1)$$

Here,  $\delta$  denotes the Dirac delta-function and  $P_0, P_1$  are the powers of the sources  $J_0, J_1$ , respectively. Authors assume that there are no sound sources in the medium up to the moment  $t = 0$ . The point receiver located at the point  $\mathbf{O}$ . In the case of [1], the source was point and isotropic ( $J = J_0$ ). Thus, the solution of the initial-boundary value problem:

$$I(\mathbf{O}, \mathbf{k}, t) = I_1(\mathbf{O}, \mathbf{k}, t) = \frac{J_1}{|\Omega|} \exp(-\mu ct) \sigma \left( \frac{ct}{2} \mathbf{k} \right) \frac{1}{t}. \quad (2)$$

Redefined  $\mathbf{r} = ct\mathbf{k}/2$  and received signal  $\bar{I}(\mathbf{r}, t) := I(\mathbf{O}, \mathbf{k}, t)$  in the equation (2) authors obtain the relation for determining the coefficient of volume scattering [1]:

$$\sigma(\mathbf{r}) = \frac{|\Omega| t \exp(\mu ct)}{P_0} \bar{I}(\mathbf{r}, t), \mathbf{r} \in \mathbb{R}^2. \quad (3)$$

Equation (3) is a solution of the inverse problem for determining the volume scattering coefficient  $\sigma$  in the single scattering approximation, a point source, emitted a pulse at the initial time, and in the case of a receiver concentrated at the origin.

The signal measured by the receiver in the single scattering approximation with a combined source  $J = J_0 + J_1$  takes the form of

$$I(\mathbf{O}, \mathbf{k}, t) = I_0(\mathbf{O}, \mathbf{k}, t) + I_1(\mathbf{O}, \mathbf{k}, t) = \int_0^{ct} \exp(\mu t') J_1 \left( -t' \mathbf{k}, \mathbf{k}, t - \frac{t'}{c} \right) dt' + \frac{P_0}{|\Omega|} \exp(-\mu ct) \sigma \left( \frac{ct}{2} \mathbf{k} \right) \frac{1}{t} + \frac{1}{|\Omega|} \int_0^{ct} \exp(-\mu t') \sigma(-t' \mathbf{k}) \int_0^R \int_0^{2\pi} J_1 \left( \mathbf{x} - t' \mathbf{k}, -\frac{\mathbf{x}}{|\mathbf{x}|}, t - \frac{t' + |\mathbf{x}|}{c} \right) \frac{\exp(-\mu |\mathbf{x}|)}{|\mathbf{x}|} \rho d\rho d\varphi dt', \quad (4)$$

where  $\mathbf{x} = r_0 + t' \mathbf{k} + \rho(\cos\varphi, \sin\varphi)$ , and the first term is responsible for the signal that does not undergo scattering in the medium.

The goal of the computational experiment is analyzing of the influence of the additional source  $J_1$  in the solution of the remote sensing problem which is obtained in the case of a point isotropic source. From the practical point of view, authors consider the problem of remote sensing in the ocean. By scattering coefficient  $\sigma$ , they simulate biological objects (plankton clusters, schools of fish).  $J_1$  describe an active source in the ocean (acoustic buoy, autonomous unmanned vehicle or submarine).

The received signal is calculated by the formula (4). Then the inverse problem of determining the coefficient of volume scattering is solved as equation (3). To solve the problem of numerical integration, the Monte Carlo method is used with the number of nodes equal to  $N$ . The volume scattering coefficient  $\sigma$  is set as

$$\sigma(y_1, y_2) = \begin{cases} 0.04, & \text{if } \sqrt{(y_1 - 40)^2 + (y_2 + 40)^2} < 10 \\ 0.02, & \text{if } \sqrt{(y_1 - 40)^2 + (y_2 - 40)^2} < 20 \\ 0.01, & \text{else.} \end{cases}$$

Numerical experiments are carried out with the following sensing parameters:  $\mu = 0.9[\text{m}^{-1}]$ ,  $c = 1500[\text{m/s}]$ ,  $x_1 \in (-80, 80)[\text{m}]$ ,  $x_2 \in [0, 40][\text{m}]$ ,  $P_0 = 1$ ,  $P_1 = \frac{10^{-4}}{\pi R^2}$ ,  $R = 15[\text{m}]$ ,  $r_0 = (0; 56)$ .

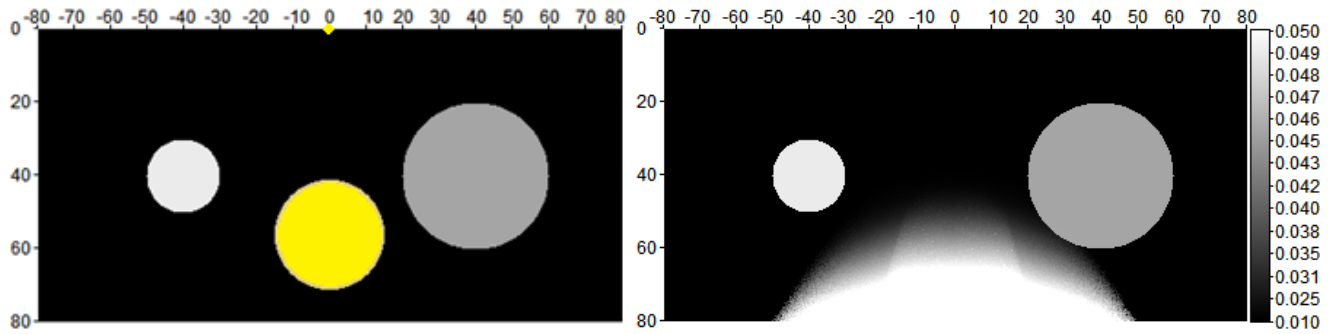


Fig. 1 Distribution of volume scattering coefficient. Left: exact solution and position of sources (yellow color). Right: recovered function  $\sigma$ .

## References

- [1] Vornovskikh P. A., Sushchenko A. A. Remote sensing problem with multiple scattering effect // Proceedings of SPIE - The International Society for Optical Engineering. 2017. N 10466 (104661Y).